Hierarchic Control Architecture for Intelligent Structures

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A new hierarchic control architecture is presented for flexible structures with widely distributed actuators and sensors. The control arrangement is a two-level combination of a centralized global controller and a set of distributed residual controllers. The global controller governs the overall motion of the structure using a reduced-order model of the finite element system. The residual controllers aggregate and pass information up to the global controller. The residual controllers also regulate the local deviations between the system and estimates obtained from the global model. A method for calculating the global and residual models and their associated controllers is developed. The coupling between these two structural models is also discussed, and some techniques for minimizing its effects are given. Finally, a simple example demonstrating the advantages of this design approach is presented.

Nomenclature

A_g, A_r = coefficient matrices
C = condensation transformation matrix
e = vector of residuals
$F_{(\cdot)}$ = feedback gain matrix on $(\cdot)$
K = finite element model stiffness matrix
M = finite element model mass matrix
m_l = mass within a local region of the system
n = number of degrees of freedom in the condensed finite element model of system
n_e = number of finite control elements
n_l = number of modes of the global model retained in alternative interpolation method 1
Q = generalized forces in the finite element model
q = vector of finite element model degrees of freedom
q_{rm} = vector of residual degrees of freedom in alternative interpolation method 1
R_{xx}, R_{uu} = linear quadratic regulator state and control weighting matrices
S = force distribution matrix
T_g = interpolation matrix for global degrees of freedom
T_r = interpolation matrix for residual degrees of freedom
u = vector of control inputs
u_e = vector of control inputs due to residual feedback
u_r = vector of control inputs due to residual feedback, after spatial filtering
W = positive definite weighting matrix
x = finite element model state vector
β = local control parameter
ξ_0 = modal degrees of freedom of the finite element model
ξ_g = modal degrees of freedom of the global model
ρ = global control weighting parameter
Φ = matrix of system eigenvectors
ϕ_0 = mode shapes of the finite element model
ϕ_g = mode shapes of the global model in the global coordinate system
Ψ = finite element model control influence matrix

Subscripts

$(\cdot)_{eval}$ = evaluation model version of $(\cdot)$
$(\cdot)_{cond}$ = condensed model version of $(\cdot)$
$(\cdot)_g$ = global model version of $(\cdot)$
$(\cdot)_r, (\cdot)_e$ = residual model version of $(\cdot)$
$(\cdot)_m, (\cdot)_s$ = master, slave coordinate version of $(\cdot)$
$(\cdot)_{gg}, \ldots, (\cdot)_{rr}$ = transformed system matrices for M, K, or F

Superscripts

$(\cdot)^g$ = global values of $(\cdot)$, used for the linear quadratic regulator weighting matrices
$(\cdot)^l, (\cdot)^u$ = lower and upper in frequency set of $(\cdot)$, respectively

I. Introduction

THERE has been a great deal of interest in the active control of structures in the past decade.[1-3] The objective of eliminating the vibratory motion of a structure is recognized as being particularly difficult in part because of the physical characteristics of the plant, principally that a structure will tend to be lightly damped and modally dense. As the performance requirements for precision control of devices are made more stringent, the control bandwidth is expanded and the flexible modes will have a significant influence on performance objectives.

Coupled with the difficulties presented by the characteristics of the structure are numerous problems with the implementation of the control law. The most common control designs

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use a central computer to perform the compensation. Some typical approaches include the use of output feedback gains directly on the measurement,[4,5] or a full- or reduced-order estimator driven by the measurements in conjunction with full-state feedback.[6,7] Many of these techniques were originally designed for the case in which the number of sensors n_s and actuators n_a is small relative to the plant dimension. One centralized technique that has received much attention recently is the independent modal-space control (IMSC) developed by Meirovitch.[8] This control employs a feedback of the modal coordinates and is designed so that the closed-loop modal equations remain independent. Implementation of this control law requires at least as many actuators as controlled modes,[8] though attempts have been made to modify the feedback to employ fewer actuators.[9]

Each of these techniques require that the central computer receive all measurements and compute every control command, therefore, there is a major difficulty with the communication demands. Also, the number of computations per cycle for a centralized output feedback controller is of the order of $n_a \times n_s$. For a dynamic compensator of order n_{dc}, both a state estimate update and a computation of order $n_a \times n_{dc}$ are required. If the controller's bandwidth is a multiple of the highest frequency to be controlled, which varies linearly or as the square of the number of modes in the structural model, then, assuming there are n modeled modes, the computational requirements of these centralized schemes increase at a rate between n^3 and n^4, which is clearly quite prohibitive for any large-scale system.

In contrast to these techniques, which are aimed at controlling structural vibrations with only a few actuators and sensors, in this research it is assumed that, as in many examples of surface control of optical devices, there are hundreds or thousands of these devices on the structure. In this case, efficiently handling the amount of information available to the controller becomes an important issue. The aim of the hierarchic architecture discussed in this paper is to take advantage of the number of actuators and sensors and to distribute the control function between two types of controllers. This will reduce both the computational burden and the input/output requirements of any one processor. This is done in a way that complements the dynamic behavior of the plant, so that the processing is distributed to reflect the physical distribution of the information flow in the structure.

The control architecture is specifically developed for intelligent structures, which have many densely packed actuators and sensors. These structures are made feasible by the development of small piezoelectric and electrostrictive actuators and other forms of sensors that can be directly embedded in a composite structure.[10-12] In fact, it is explicitly assumed in this analysis that there are enough actuators on the structure that a model condensed to a dimension of the number of actuators would still accurately reflect the dynamics of the structure, at least for the lower frequency modes. A typical example of the applicability of this control architecture would be the shape control of a mirror whose back surface is covered with these piezoelectric actuators and sensors. The hierarchic architecture developed here consists of two distinct levels of control. The lower control level has many regional processors that interface with the actuators and sensors in separate regions of the structure and are, in some sense, independent. The function of the second level is to coordinate the lower level controllers and perform the global control functions.

Recently, there have been many control techniques developed as alternatives to the centralized control approach. One solution to the problems posed by the centralized design would be to distribute the control effort among several processors, thereby making the approach more parallel in nature. This is usually implemented as a decentralized controller that consists of several isolated regional controllers (in the sense of knowledge and influence) that are distributed throughout the structure. The necessary and sufficient conditions for the existence of a local output feedback law that stabilizes a given system were originally developed by Wang and Davison.[13] Many of the decentralized and multilevel control techniques are summarized by Sandell et al.[14] The published decentralized control approaches typically fall into one of two catagories. One design approach uses a model of the entire structure and then imposes a constraint on the information available to be fed back. The second approach decomposes the system into coupled subsystems, and the controllers are designed by ignoring the interactions between subsystems. For example, West-Vukovich et al.[15] restricted the information available to each actuator and designed a collocated PID controller for a flexible plate model. Joshi[16] also imposed a collocated feedback restriction and investigated the robustness and performance of different types of feedback. Silverberg[17] introduced the decentralized uniform damping control law, which was shown to be a first-order expansion of the energy weighted linear quadratic regulator (LQR). Šiljak[18] developed the notation for subsystem decomposition and the concept of connective stability for weakly linked subsystems. Ikeda and Šiljak[19] introduced more general decompositions, and Young[20] employed an overlapping decomposition technique to reduce the intersubsystem coupling and allow adjacent controllers to share information.

Other decentralized control approaches include those by Bennet and Lindberg[21] and Iftar and Ozguner.[22] Both developed decentralized control approaches that depend on the diagonal dominance of the plant's transfer function. Bernstein[23] designed a decentralized controller using the optimal projection equations. His approach was to perform a sequential optimization of isolated fixed-order dynamic controllers that act as the central controller for the plant with all other controllers in place. Young[24] used an approach called controlled component synthesis, which is important to this discussion because the isolated controllers were designed using specific information from the structural model. The isolated subsystems (components) were modeled and controlled at the component level in a manner that was analogous to the familiar component mode synthesis[25] and then connected together to obtain the full controlled structure.

The main difficulty with any decentralized approach is that there typically is very little control authority to handle the longer wavelength motions. Since each regional controller can only measure and influence the adjacent portions of the structure and is unaware of the gross motions of the entire body, it is not well suited for designs that have performance objectives for the overall structure. Most of the techniques that have been discussed are better suited for the design of controllers for systems that can be written in terms of lightly coupled subsystems. In this case, the interactions are small enough that they can be ignored during the design phase and the coupling in the overall structure will only have a small influence on the closed-loop subsystems. An argument about the overall stability can then be made using either the input-output approach[26] or the connective stability from Šiljak.[18] For example, this approach is employed in Ref. 27 to control the segments of a telescope. However, for strongly coupled structures, one cannot arbitraily break up the structure into subsystems and expect these decentralized control designs to stabilize the overall structure. The hierarchic control approach developed in this paper employs a decentralized controller at the lower level, but the design does not require that the structure spatially decompose into weakly linked subsystems. Instead, the architecture decomposes the structure into weakly linked frequency subspaces.

A commonly employed second alternative to the centralized control approach is to use multilevel controllers. Examples exist that implement these hierarchic controllers on large-scale systems such as traffic patterns or power stations.[28-30] A different hierarchic approach[31] is essentially a decentralized controller based on succesive loop closure that employs a second level to eliminate some of the spillover effects. A similar approach, using global and local controllers, was developed by

Šiljak[18] and used by Pitman and Ahmadian.[32] The local controllers were designed for performance using the decomposition approach discussed previously, and the global controller was designed to minimize the coupling between the subsystems. The resulting architecture requires that the central controller have access to all of the state information at the local level. This differs from the hierarchic architecture developed in this paper since the global controller in our design is more performance oriented and only requires an aggregate of the local states to perform the control, and so it is more suitable for very large-scale systems.

Craig and Hale[33] and Su and Craig[34] developed a structural model-reduction technique, which is interesting in terms of this work because the reduction was done in a manner that complemented the structure of the plant. Krylov vectors were introduced to develop a good reduced-order model that eliminates the control and observation spillover to the residual system. Yam et al.[35] developed a similar approach based on an aggregation using sensor and actuator influence functions. These approaches closely mirror the aggregation to the global model performed in our hierarchic design.

Another important technique, which also employs a division in the control effort, is the high authority/low authority (HAC/LAC) control design.[36,37] The degree of authority is a measure of the influence of the controller on the structure. The LAC supplements the damping of the structure by using collocated rate feedback, thereby reducing the possibility of destabilization due to spillover.[38] The HAC loop is designed to meet the performance specifications for the plant. An important characteristic of this architecture is that there is no coordination between the control effort at the two levels. The input/output requirements of this approach are similar to those for a single central controller.

The hierarchic control architecture in this paper is an improvement over other techniques because it allows the designer to perform independent control development at the global and local levels, not just successive loop closure. Although not explicitly shown in this paper, it also allows for a far more sophisticated inner loop.[39] The key benefit, though, is in the way that the measurements are processed. The information is independently aggregated by the local controllers at the lower level and then only a reduced subset is passed to the central processor. Finally, this method provides virtually equivalent performance to the LQR, but it is implemented in a more computationally efficient manner. In this way, some of the benefits (parallel in nature) but not all of the disadvantages (retains a large global control authority) of both the centralized and decentralized schemes have been incorporated into one architecture.

This paper details the development and evaluation of a two-level hierarchic control methodology for implementation in flexible structure control.[40] The methodology outlines the division of control between global and regional processors and specifies the functions performed at the two different levels. Techniques to reduce the dynamic coupling between the two control levels are then discussed, as are algorithms for developing the two controllers. Finally, a simple example of the control of a beam in bending is presented.

II. Architecture

The fundamental idea behind the hierarchic control formulation presented here is to develop a parallelism between the element/global hierarchy of a structural model and the regional/global hierarchy of the active control. In structural dynamic analysis, it is not uncommon to model elements or components of the structure in detail and then, by techniques such as component mode analysis or condensation, extract from the detailed local models that information that governs the overall or global motion. In a parallel manner, local controllers should be able to regulate the detailed behavior within a component or region while the coordinating effort of a global controller regulates the gross or overall motion. From a physical perspective, just as short wavelength disturbances are propagated in a structure locally, the short wavelength control is performed by the local controllers. As it is possible for long wavelength disturbances to develop into modes, there is a global controller for control of the overall motion.

The point of departure of the structural modeling is a finite dimensional model of the structure. The motion of this evaluation model is represented by the degrees of freedom q_{eval}. The first step in the analysis is to condense this model to one in which the structure is modeled by a subset of q_{eval}, namely q_m, the master degrees of freedom, each of which is associated with an actuator (and collocated sensor). Since it is assumed that the sensors and actuators are numerous, this condensation should still yield a fairly accurate model. This process will be outlined in Sec. III. As a result of this procedure, both the measurement and control influence matrices in the condensed model (to be introduced in Sec. IV) are of full rank.

The next step is to divide the structure into finite control elements, each of which contain several of the master degrees of freedom q_m. This coarse or global model has as its coordinates the global degrees of freedom q_g at discrete node points located at the boundaries of the finite control elements. The global displacements are related to the degrees of freedom of the condensed finite dimensional model by the element interpolation functions (Fig. 1) with q representing the master degrees of freedom. Likewise, there are forces Q in the condensed model associated with the degrees of freedom q_m, and global forces Q_g associated with the global degrees of freedom.

The corresponding division of control functions in a two-level hierarchic controller is outlined in Fig. 2. The objective of the global controller is to control the overall behavior of the structure, based on the global states $x_g^T = [q_g^T \dot{q}_g^T]$. The three basic tasks that are involved in implementing the global control are shown in Fig. 2: the measurement aggregation, which reduces the measurements q_m into measurements of the states in the global model q_g; the computation by the global controller of the global control commands Q_g based on the global states; and the distribution of the global control, which calculates the physical control forces u_g to be applied to the structure. Note that both the measurement aggregation and the control distribution require communication with the structure through the local controllers, so that these functions are performed cooperatively between the global and regional processors.

There are a number of regional controllers, each associated with a finite control element. Each operates on the residual e, the difference of the actual local measurements q_m and the

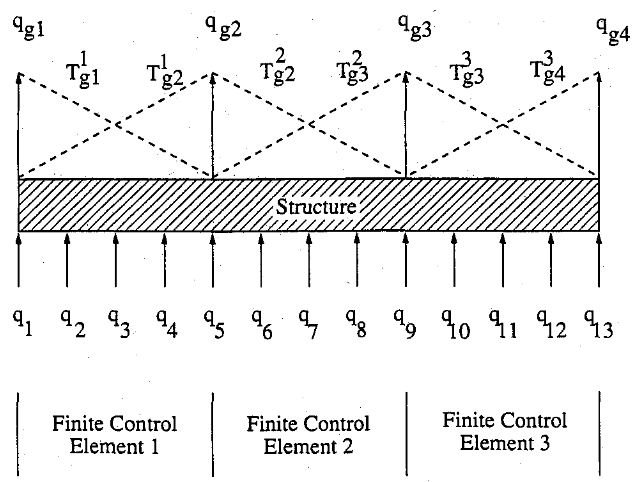

Fig. 1 Representation of a condensed model of the structure by a coarse or global finite element model denoted by $(\cdot)_g$. The shape functions $T^i_{g_j}$ relate the nodes of the condensed model in element (i) to global node (j).

interpolation of the local estimates from the global states q_g. The specific objective of the regional controller is to perform inner loop compensation on this residual in order to force the structure to track the behavior expected by the global model. The residual controller calculates control forces u_e, which are then spatially filtered to ensure that the global modes are not excited by the residual control. It will be seen that this spatial filtering is accomplished more easily by a cooperative effort between the global and regional processors than by the regional processors alone. The resulting control command u_r is added to the global control u_g to form the total control command u to the structure.

III. Model Condensation

As outlined in the previous section, the first step in the analysis is to perform a condensation of the evaluation model. When the inertia forces of a subset of the states of the model can be considered to be more important than the rest, it is possible to reduce the order of the model through a condensation procedure without incurring significant errors. This technique is commonly known as both mass condensation[41] and Guyan reduction.[42] Irons[43] and Archer[44] recommended the rotational degrees of freedom of a beam or a plate as good examples of coordinates that can be condensed out (i.e., the slave coordinates). The condensation procedure assumes a static equilibrium with respect to the slave coordinates, or equivalently, that the potential energy of the system has a minimum with respect to the slave displacements.

The states of the evaluation model can be reordered and grouped as master and slave to yield

$$q_{\text{eval}} = \begin{bmatrix} q_m \\ q_s \end{bmatrix} \tag{1}$$

where both the mass and stiffness matrices of the evaluation model are partitioned as

$$K_{\text{eval}} = \begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix} \tag{2}$$

Taking the potential energy of the system to be stationary with respect to the slave coordinates yields a transformation matrix

$$C = \begin{bmatrix} I \\ -K_{ss}^{-1}K_{sm} \end{bmatrix} \tag{3}$$

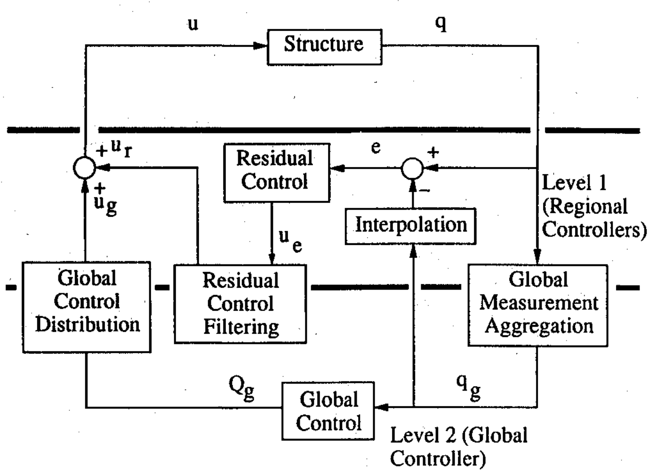

Fig. 2 Division of control functions in a two-level hierarchic controller. The measurement aggregation, the control distribution, and the residual filtering tasks are shared between the two levels. The measurements q correspond to the degrees of freedom of the condensed model.

which expresses the full set of states in Eq. (1) in terms of the master degrees of freedom, where the inverse of K_{ss} is assumed to exist since the master degrees of freedom contain sufficient information to model the rigid-body modes of the plant. The resulting condensed mass and stiffness matrices are

$$K_{\text{cond}} = C^T K_{\text{eval}} C = K_{mm} - K_{ms}K_{ss}^{-1}K_{sm} \tag{4}$$

$$M_{\text{cond}} = C^T M_{\text{eval}} C = M_{mm} - K_{ms}K_{ss}^{-1}M_{sm} - M_{ms}K_{ss}^{-1}K_{sm} + K_{ms}K_{ss}^{-1}M_{ss}K_{ss}^{-1}K_{sm} \tag{5}$$

This procedure can be used to condense out the unactuated states of the evaluation model

$$\begin{bmatrix} M_{mm} & M_{ms} \\ M_{sm} & M_{ss} \end{bmatrix} \begin{bmatrix} \ddot{q}_m \\ \ddot{q}_s \end{bmatrix} + \begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix} \begin{bmatrix} q_m \\ q_s \end{bmatrix} = \begin{bmatrix} \Psi \\ 0 \end{bmatrix} u \tag{6}$$

where it is assumed that only the master states are actuated directly. Employing the techniques given earlier and noting that

$$C^T \begin{bmatrix} \Psi \\ 0 \end{bmatrix} = \Psi \tag{7}$$

the condensed dynamics of the structure can then be represented by an undamped finite dimensional model of the form

$$M_{\text{cond}}\ddot{q}_m + K_{\text{cond}}q_m = \Psi(u_g + u_r) \tag{8}$$

where $q_m \in R^n$ is the vector of generalized coordinates of the condensed model, $u \in R^n$ is the vector of control inputs, $M_{\text{cond}} \in R^{n\times n}$ is a symmetric, positive definite mass matrix, $K_{\text{cond}} \in R^{n\times n}$ is a symmetric, positive semidefinite stiffness matrix, and $\Psi \in R^{n\times n}$ is the control influence matrix. Note that, because of the condensation of the evaluation model, complete measurements of all states (q_m and $\dot{q}_m$) are available, and the system has as many nonredundant actuators as generalized coordinates (i.e., Ψ is square and full rank). The subscripts $(\cdot)_{\text{cond}}$ and $(\cdot)_m$ are now dropped for notational convenience.

IV. Defining the Global and Residual Coordinates

In view of the architectural objectives, it is necessary to derive a partitioning of the structural dynamic model that achieves the greatest possible dynamic decoupling of the global and residual models. The first step is a suitable definition of the global and residual coordinates. The condensed dynamics of the structure are first represented in the form of Eq. (8). The coarser or global model is assumed to have n_g degrees of freedom and associated interpolation functions, represented by the matrix $T_g \in R^{n\times n_g}$. The actual displacements are then a sum of the interpolated displacements of the global model and a vector of n residuals

$$q = T_g q_g + e \tag{9}$$

where $q_g \in R^{n_g}$, and $e \in R^n$. For any set of actual displacements q and assumed interpolation functions T_g, q_g can be defined to minimize a weighted quadratic of the residual error

$$J = e^T W e \tag{10}$$

where W is an appropriately selected positive definite weighting matrix. Although there are n entries in e, there are only $n_r = n - n_g$ independent degrees of freedom designated q_r, so that e may be expressed as

$$e = T_r q_r \tag{11}$$

where $T_r \in R^{n \times n_r}$ is yet to be determined. The actual displacements can then be expressed in terms of the global and residual coordinates as

$$q = T_g q_g + T_r q_r = [T_g \quad T_r] \begin{bmatrix} q_g \\ q_r \end{bmatrix} \tag{12}$$

The condition that q_g is defined to minimize Eq. (10) establishes that the global degrees of freedom are related to the condensed degrees of freedom q by

$$q_g = (T_g^T W T_g)^{-1} T_g^T W q = T_g^{-L} q \tag{13}$$

where the superscript $(\cdot)^{-L}$ denotes the left pseudoinverse of $(\cdot)$. The global degrees of freedom determined by Eq. (13) will produce the best weighted least squares fit to the actual displacements. By substituting Eq. (12) into Eq. (13), or equivalently, using the optimal projection theorem,[45] one obtains

$$T_g^T W T_r = 0 \tag{14}$$

indicating that the two subspaces spanned by the columns of the matrices T_g and T_r are orthogonal with respect to the weighting matrix W. The substitution of Eq. (13) into Eq. (9) yields the following expression for the residuals e in terms of the displacements q:

$$e = (I - T_g T_g^{-L}) q \tag{15}$$

The expression for the residual degrees of freedom q_r in terms of q is given by

$$q_r = T_r^{-L} q \tag{16}$$

T_r may be determined by a number of methods, e.g., by performing a Gram-Schmidt orthogonalization on any set of n_r columns that, when combined with the columns of T_g, form a linearly independent set. Note, however, that the columns of T_r are not unique; the only requirement is that they span the subspace that is orthogonal (with respect to the weighting matrix W) to the column space of T_g. Furthermore, it will be seen that it is not necessary to calculate T_r in order to develop the hierarchic controller.

V. Decoupling the Control/Structural Model

Having established the relationships among the various representations of the degrees of freedom q, q_r, q_g, and e, these relationships can be used to analyze the extent to which the dynamics of the global and residual degrees of freedoms can be decoupled. To examine the subspace coupling, Eq. (12) is substituted into Eq. (8) and premultiplied by $[T_g \ T_r]^T$ to give

$$[T_g \quad T_r]^T M [T_g \quad T_r] \begin{bmatrix} \ddot{q}_g \\ \ddot{q}_r \end{bmatrix} + [T_g \quad T_r]^T K [T_g \quad T_r] \begin{bmatrix} q_g \\ q_r \end{bmatrix}$$
$$= [T_g \quad T_r]^T \Psi (u_g + u_r) \tag{17}$$

The degree of coupling in each of the three terms of Eq. (17) can now be examined. The first step is to expand the control influence terms on the right side. The inputs u_g are the physical forces based on the n_g commanded global forces Q_g, each associated with a global displacement. The global forces Q_g are distributed into the physical inputs u_g according to a relation of the form

$$u_g = S_g Q_g \tag{18}$$

where S_g is the global force distribution matrix. The residual control forces will similarly be commanded (within a finite control element) by the residual controller. The residual forces will be distributed as

$$u_r = S_e Q_e \tag{19}$$

where Q_e is the vector of the n commanded forces associated with the residuals e, which are, of course, related to the residual degrees of freedom q_r through Eq. (11). The assumed form of the feedback laws for the two levels is

$$Q_g = -F_g q_g - F_{\dot{g}} \dot{q}_g \tag{20}$$

$$Q_e = -F_e e - F_{\dot{e}} \dot{e} \tag{21}$$

The nature of the coupling of the global and residual degrees of freedom can be evaluated by substituting Eq. (11), (18), (20), (19), and (21) into Eq. (17) giving

$$\begin{bmatrix} M_{gg} & M_{gr} \\ M_{rg} & M_{rr} \end{bmatrix} \begin{bmatrix} \ddot{q}_g \\ \ddot{q}_r \end{bmatrix} + \begin{bmatrix} K_{gg} & K_{gr} \\ K_{rg} & K_{rr} \end{bmatrix} \begin{bmatrix} q_g \\ q_r \end{bmatrix}$$
$$= -\begin{bmatrix} T_g^T \Psi S_g F_g & T_g^T \Psi S_e F_e T_r \\ T_r^T \Psi S_g F_g & T_r^T \Psi S_e F_e T_r \end{bmatrix} \begin{bmatrix} q_g \\ q_r \end{bmatrix}$$
$$-\begin{bmatrix} T_g^T \Psi S_g F_{\dot{g}} & T_g^T \Psi S_e F_{\dot{e}} T_r \\ T_r^T \Psi S_g F_{\dot{g}} & T_r^T \Psi S_e F_{\dot{e}} T_r \end{bmatrix} \begin{bmatrix} \dot{q}_g \\ \dot{q}_r \end{bmatrix}$$
$$= -\begin{bmatrix} F_{gg} & F_{gr} \\ F_{rg} & F_{rr} \end{bmatrix} \begin{bmatrix} q_g \\ q_r \end{bmatrix} - \begin{bmatrix} F_{g\dot{g}} & F_{g\dot{r}} \\ F_{r\dot{g}} & F_{r\dot{r}} \end{bmatrix} \begin{bmatrix} \dot{q}_g \\ \dot{q}_r \end{bmatrix} \tag{22}$$

Equation (22) should now be examined in light of the stated objective of minimizing the coupling between the global and residual subsystems. There are four matrices in Eq. (22) that must be decoupled, but the two control influence matrices on the right side have the same internal structure.

By comparing the off-diagonal blocks of the transformed mass and stiffness matrices with Eq. (14), it is clear that the appropriate choice of W in the definition of q_g can cause either the off-diagonal terms of the mass or the stiffness matrices to be driven to zero. Choosing W to be M would inertially decouple the residuals, whereas choosing W to be K would elastically decouple them. In that M is positive definite, it is the preferred choice. Furthermore, the selection of W equal to M would cause Eq. 14 to resemble the primary orthogonality relation of the dynamic system.

The choice of the defining weighting matrix W to be M identically sets M_{gr} and M_{rg} equal to zero, thereby block diagonalizing the transformed mass matrix of Eq. (22). However, to some extent, it also diagonalizes the stiffness matrix. In the ideal case, the n_g modes of the condensed system can be exactly represented by q_g coordinates and their associated shape functions T_g, such that all of the modes can be given by

$$\Phi = [T_g \quad T_r] \begin{bmatrix} A_g & 0 \\ 0 & A_r \end{bmatrix} \tag{23}$$

where Φ is an $n \times n$ matrix of eigenvectors of M and K, A_g is an $n_g \times n_g$ matrix of coefficients, and A_r is an $n_r \times n_r$ matrix of coefficients. In such a case, T_g contains the exact shapes of n_g modes and

$$T_g^T K T_r = K_{gr} = 0 \tag{24}$$

In the less ideal case when T_g closely approximates a subset of n_g modes of the condensed system, the off-diagonal terms of the stiffness matrix K_{gr} are small compared to K_{gg} and K_{rr}. Clearly, this is a desirable property to have in the shape functions.

The matrices on the right side of Eq. (22) can be block diagonalized by a proper choice of the constitutive matrices. Examining the lower left term F_{rg} (or equivalently, $F_{r\dot g}$), which expresses the spillover of the global control into the residual model, we have

$$F_{rg} = T_r^T \Psi S_g F_g \tag{25}$$

T_r has been established by the initial choice of shape functions and Eq. (14), and Ψ by the location and type of the actuators. One would prefer not to place restrictions on F_g or $F_{\dot g}$ that would complicate the subsequent control design synthesis. Therefore, only the form of the global force distribution matrix S_g can be specified so as to drive this term to zero. In the case of Ψ invertible, a normalized choice is

$$S_g = \Psi^{-1} M T_g (T_g^T M T_g)^{-1} = \Psi^{-1} T_g^{-LT} \tag{26}$$

where the superscript $(\cdot)^{-LT}$ denotes the transpose of the left pseudoinverse. Substituting Eq. (26) into Eq. (18) gives

$$\Psi u_g = T_g^{-LT} Q_g \tag{27}$$

The implication of this equation is that the n_g global control forces are distributed to the n actuators using the same shape functions and weighting matrices as are used in aggregating the information from the n sensors to form the n_g global displacements in Eq. (13). This symmetry of global state aggregation and global control distribution results from the requirement that the global and residual degrees of freedom be uncoupled, both in the inertial term in the dynamics equation and in the feedback term.

The upper right entry F_{gr}, which expresses the spillover of the residual control into the global modes, is

$$F_{gr} = T_g^T \Psi S_e F_e T_r \tag{28}$$

Again, T_g, T_r, and Ψ have already been prescribed, and so a proper choice of S_e coupled with a form of F_e must be made to drive this term to zero. One normalized choice is

$$S_e = \Psi^{-1} M T_r (T_r^T M T_r)^{-1} T_r^T = \Psi^{-1} T_r^{-LT} T_r^T \tag{29}$$

At this point, all of the coupling terms of Eq. (22) have been examined. The choice of W to be M in Eq. (10) plus the expressions in Eq. (26) and (29) reduce Eq. (22) to

$$\begin{bmatrix} M_{gg} & 0 \\ 0 & M_{rr} \end{bmatrix} \begin{bmatrix} \ddot q_g \\ \ddot q_r \end{bmatrix} + \begin{bmatrix} K_{gg} & K_{gr} \\ K_{rg} & K_{rr} \end{bmatrix} \begin{bmatrix} q_g \\ q_r \end{bmatrix} = - \begin{bmatrix} F_g & 0 \\ 0 & T_r^T F_e T_r \end{bmatrix} \begin{bmatrix} q_g \\ q_r \end{bmatrix} - \begin{bmatrix} F_{\dot g} & 0 \\ 0 & T_r^T F_{\dot e} T_r \end{bmatrix} \begin{bmatrix} \dot q_g \\ \dot q_r \end{bmatrix} \tag{30}$$

where the mass and control influence matrices are completely uncoupled and the stiffness matrix is uncoupled to the extent that T_g models n_g of the modes of the condensed finite dimensional system.

The effect of the global and residual control on the condensed finite dimensional model can also be determined by substituting Eq. (13), (16), (18-21), (26), and (29) into Eq. (8) to obtain

$$M\ddot q + K = -\{T_g^{-LT} F_g T_g^{-L} + T_r^{-LT} T_r^T F_e T_r T_r^{-L}\} q K q - \{T_g^{-LT} F_{\dot g} T_g^{-L} + T_r^{-LT} T_r^T F_{\dot e} T_r T_r^{-L}\} \dot q \tag{31}$$

The physics of the control decoupling can be seen by examining the first of the two terms on the right side of Eq. (31). Examining the terms in detail, the role of T_g^{-L} is to aggregate or average the elements of q to determine the global coordinates q_g. Multiplying by the matrix F_g produces the global forces Q_g (due to displacement feedback), which are then distributed by T_g^{-LT}. The roles of T_r, T_r^{-L}, and F_e are similar in the residual subspace. T_r^{-L} aggregates the measurements to form the residual coordinates q_r, and then T_r redistributes them to form the vector of the residuals e. F_e then multiplies e to produce the residual forces Q_e. Finally, $T_r^{-LT} T_r^T$ acts as a spatial filter, which produces forces that affect only the residual degrees of freedom q_r. This may be seen by use of the identity

$$T_r T_r^{-L} = I - T_g T_g^{-L} \tag{32}$$

That is, $T_r^{-LT} T_r^T$ is a projection matrix that, when multiplying Q_e, eliminates those components of force that affect the q_g subspace.

An alternative way to write the first term on the right side of Eq. (31) is to use Eq. (32) to give

$$\Psi u_q = -\{T_g^{-LT} F_g T_g^{-L} + (I - T_g^{-LT} T_g^T) F_e (I - T_g T_g^{-L})\} q \tag{33}$$

which suggests the architecture in Fig. 3. (u_q is that part of the control u due to feedback of q, with a similar term due to the feedback of $\dot q$.) Loop "o" is the process by which the global motion is filtered from the overall motion to form the residual (observation filtering). Loop "c" is the process by which the global component is filtered out of the residual commands (control filtering). Also shown in the figure is the distribution of the computing resources between global and residual controllers. Note that, in this architecture, there is never a need to explicitly calculate T_r or to determine q_r. Also, note that the processing is performed in such a way that the residual processors carry out most of the calculations in parallel at the local level, with the global processor performing control only on the global states q_g, as well as communicating this information to the residual controllers. It is interesting to note that Eq. (31) and (33) show that the combination of the two sets of gains and the control architecture generate full gain matrices that have a specified internal structure.

The assumption that the structure is intelligent implies that the controller must govern many sensors and actuators. The advantage of this hierarchic control architecture is that the work load is separated between the two levels. The measurements are aggregated before any communication with the global processor is performed. This step can be implemented efficiently because the local controllers are distributed in such a manner as to complement the dynamic behavior of the structure. The work at level 1 is also done in parallel, which is the most efficient way to organize the effort. This aggregation procedure is split between the two levels so that each controller is only required to act on the information that it has directly available.

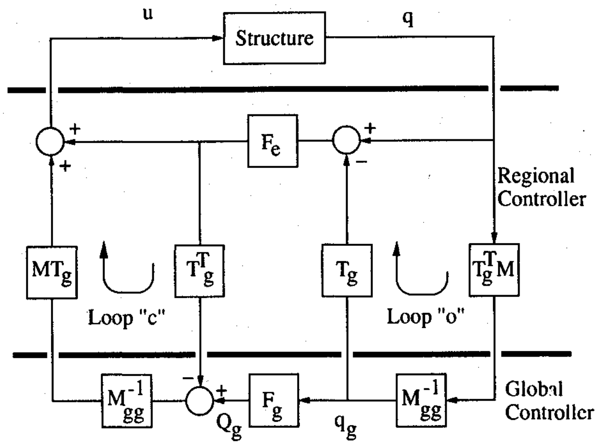

Fig. 3 Hierarchic control architecture, clearly showing the division of control effort between the two levels.

Another important issue to consider is that of spillover. Through the assumptions made about the plant, the observation and control spillover can be reduced in the hierarchic design by adding the two filtering arms in the loops. It is these two extra paths that differentiate this architecture from a standard HAC/LAC design. Since the global and residual subsystems are still dynamically coupled, spillover will exist. However, it is possible to modify the design to reduce the influence of this effect (Sec. VI). The resulting architecture then allows for independent control design at each of the two levels.

The transformed and untransformed state representations [Eq. (30) and (31), respectively], which include the coupled residual and global control, have now been developed. The two remaining key questions are: (1) how to choose the shape function T_g so as to minimize the stiffness matrix coupling, and (2) how to synthesize the residual control to be regional in form.

VI. Reduction of the Dynamic Coupling

A key issue that must be recognized at this point is that the number of global degrees of freedom and the shape functions must be selected to provide a good model of the important modes of both the open- and closed-loop systems. For the vibration control work performed here, this will typically be the set of lower frequency modes. The number of global degrees of freedom must be selected so that the global controller has sufficient authority to place the important closed-loop poles at the desired frequencies and damping ratios. The importance of a particular mode to the global cost function can be analyzed using the modal cost analysis by Skelton et al.[7] In forming Eq. (12), several alternatives for the choice of T_g have been developed with the aim of minimizing the K_{gr} stiffness coupling term between the two subsystems.

Alternative 1

One alternative is to keep fewer modes in the global model than there are global degrees of freedom. This follows directly from common experience with finite element models in that, at best, the designer can have confidence in only the lower half of the modeled modes and, in general, the number is usually significantly less than this. In the model of the structure, which only includes shapes built up from T_g, some of the modes of the condensed finite dimensional system will be well modeled, but others (generally the upper frequency modes) will be modeled more poorly. Since, in general, the degree to which K_{gr} approaches zero depends on the ability of T_g to represent a subset of the modes of the condensed finite dimensional model, it may be desirable to reduce the number of modes retained in the global model n_l to a value less than the number of global degrees of freedom n_g.

In this first alternative, let the global displacements q_g be expanded in terms of the modes of the global model ϕ_g as

$$q_g = \begin{bmatrix} \phi_g^l & \phi_g^u \end{bmatrix} \begin{bmatrix} \xi_g^l \\ \xi_g^u \end{bmatrix} \tag{34}$$

where $(\cdot)^l$ refers to the lower (in frequency) set, and $(\cdot)^u$ to the upper set. Substituting Eq. (34) into Eq. (9),

$$q = T_g \begin{bmatrix} \phi_g^l & \phi_g^u \end{bmatrix} \begin{bmatrix} \xi_g^l \\ \xi_g^u \end{bmatrix} + e \tag{35}$$

If only the lower modes are to be maintained in the global model, then the upper modes can be considered to form a part of the residual, so that

$$q = T_g \phi_g^l \xi_g^l + e_m = T_{gl} \xi_g^l + T_{rl} q_{rm} \tag{36}$$

where Eq. (36) now parallels Eq. (9). Minimizing the quadratic of the new residual e_m weighted by the mass matrix yields the appropriate definition of the lower global coordinates

$$\begin{aligned} \xi_g^l &= (T_{gl}^T M T_{gl})^{-1} T_{gl}^T M q \\ &= \left[(\phi_g^l)^T T_g^T M T_g (\phi_g^l) \right]^{-1} (\phi_g^l)^T T_g^T M q \end{aligned} \tag{37}$$

All of the previously derived results can now be used in this variation, with the expression for the modified interpolation functions

$$T_{gl} \equiv T_g \phi_g^l \tag{38}$$

substituted for T_g.

Alternative 2

A direct way to reduce the coupling through the stiffness term K_{gr} is to simply choose the interpolation functions to be the first n_g modes of the system. This will result in the complete decoupling of the global and residual systems. That is, the condensed degrees of freedom may be expressed as

$$q = \phi_0 \xi_0 \tag{39}$$

where ϕ_0 is the matrix whose columns are the modes of the system and ξ_0 is the vector of modal amplitudes. Assuming that n_g of these will be used to represent the global model, we have that

$$q = \phi_0^l \xi_0^l + \phi_0^u \xi_0^u = \phi_0^l \xi_0^l + e_0 \tag{40}$$

where the superscript $(\cdot)^l$ again denotes the lower set of modes, so these modes now constitute the modified interpolation functions and

$$T_{g0} \equiv \phi_0^l \tag{41}$$

Minimizing the quadratic error of e_0 weighted by the mass matrix M yields the following new definition of the global coordinates:

$$\xi_g^l = \xi_0^l = \left[(\phi_0^l)^T M (\phi_0^l) \right]^{-1} (\phi_0^l)^T M q \tag{42}$$

where the term within the inverse is diagonal if the actual normal modes are used. In this case, the global and residual subspaces are completely uncoupled since the off-diagonal terms of the stiffness matrix K_{gr} are now zero. All of the previous results are now valid, with $T_{g0} = \phi_0^l$ substituted for T_g. In this form, the hierarchic controller is reminiscent of the IMSC design discussed previously.

Using modes for the interpolation functions results in a loss of one of the implementation advantages of this technique since these modes are in general nonzero over the entire structure. By comparison, the standard interpolation functions of beam or rod finite element models are nonzero only over a few elements. Consequently, to implement this second alternative, each element controller must calculate the contribution of the local measurements to each mode of the system and then all of this information must be communicated to the central computer where it is combined to form the modal coordinates. This represents significantly more work than having each controller only calculate the contribution to the shape functions that are nonzero within its region. Thus, there is a tradeoff in terms of the degree of decoupling that is required between the two systems and the increased cost of the control calculation.

VII. Hierarchic Control Synthesis

Having derived the options for partial or total subspace decoupling, the subspace controllers must now be synthesized. The following algorithm is an outline of the steps to be followed in designing a hierarchic control system:

The choice of the element interpolation functions. As discussed in the previous section, the number of global degrees of freedom and the interpolation functions must be selected to provide a good model of the lower frequency modes and a central controller that has sufficient control authority to perform the global tasks. In the selection of the shape functions T_g, the minimization of the coupling between the global and residual models should be emphasized, and it can be checked by looking at the relative norms of the off-diagonal blocks on the left side of Eq. (22).

Global control design. Assume the stiffness coupling linking the two subsystems is zero ($K_{gr}=0$) and design the global control for the global design model

$$M_{gg}\ddot{q}_g + K_{gg}q_g = Q_g = -F_g q_g - F_{\dot{g}}\dot{q}_g \tag{43}$$

by any appropriate means. This is a simple control design because, by definition, all of the states are available to be fed back, and so any full state feedback technique can be implemented. One approach is to use a LQR, with a state vector $x_g^T = [q_g^T \quad \dot{q}_g^T]$, and reduced state and control weighting matrices

$$R_{xx}^g = \begin{bmatrix} T_g^T & 0 \\ 0 & T_g^T \end{bmatrix} R_{xx} \begin{bmatrix} T_g & 0 \\ 0 & T_g \end{bmatrix} \tag{44}$$

$$R_{uu}^g = S_g^T R_{uu} S_g \tag{45}$$

where R_{xx} and R_{uu} are the weighting matrices for the condensed finite element model (FEM).

Local control design. Design the local controller for the residual design model so that it provides good shape control as per the stated objectives of the local controller. The controller can be designed as a decentralized controller on q and then implementated as a filtered local controller on e.

Performance evaluation. The performance can then be evaluated by using Eq. (30) to measure the influence of the dynamic spillover and the degree of suboptimal behavior that is introduced by this coupling between the subsystems. If the coupling is found to be too high, then some of the alternatives provided in Sec. VI can be employed to reduce the interaction.

VIII. Example

In the following example, an application of the hierarchic control design procedure will be considered. The structure to be analyzed is a 30-node beam, (Fig. 4) which represents a simple model for many more complex systems.[46] The evaluation FEM includes both the displacement and rotational degrees of freedom at each node. This latter set of coordinates are condensed out to generate the design model with $n=30$. The beam is assumed to be uniform and undamped, with the mass per unit length, the stiffness, and the distance between nodes of the FEM all set to unity. As shown in Fig. 4, six evenly spaced global nodes are used, resulting in five finite control elements ($n_e=5$). Since there are two degrees of freedom associated with each global node, the resulting global design model has a total of 12 degrees of freedom ($n_g=12$). For this simple example, it is assumed that the displacement and velocity can be measured at each node and that there is a collocated force actuator. These assumptions are then consistent with a typical IMSC design. The shape functions T_g were designed using the same cubic polynomials employed to create the evaluation FEM.

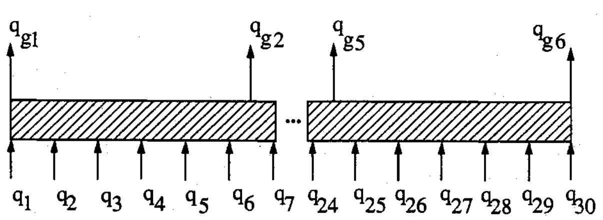

Fig. 4 30-node beam model with collocated displacement, velocity sensors, and force actuators.

The global controller is based on a LQR design. The aim was to pick a realistic control objective for the design so that the full-order state penalty includes a sum of the line-of-sight (LOS) pointing error for the two end points, the average squared displacement at each node, and the energy of the beam, each of which have a varying degree of influence on the overall closed-loop performance. The same control weighting was used for each actuator. Consequently, the condensed model LQR weighting matrices are

$$x^T R_{xx} x = \alpha_1(q_1 - q_n)^2 + \alpha_2 \sum^n q_i^2 + \alpha_3(q^T K q + \dot{q}^T M \dot{q}) \tag{46}$$

$$R_{uu} = \rho I_n \tag{47}$$

In this example, $\alpha_1=2$, $\alpha_2=1$, $\alpha_3=0.1$, and $\rho=10$ to place a relatively large weighting on the LOS and displacement penalties. The appropriate weighting matrices for the global design were then obtained using Eq. (44) and (45). The local control design employed a combination of collocated displacement and velocity feedback, which was weighted by the local mass m_l of the beam,[17] so

$$F_e = \beta^2 m_l, \qquad F_{\dot{e}} = 2\beta m_l \tag{48}$$

where the parameter $\beta=0.1$ was picked to provide a sufficient level of damping in the higher frequency modes.

With the global and local controllers defined, the closed-loop pole locations can be obtained. The results are depicted in

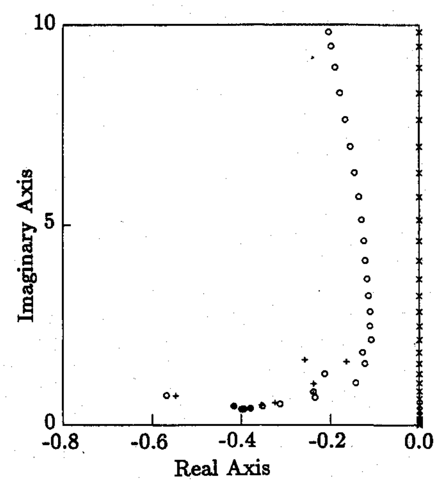

Fig. 5 Closed-loop poles when the hierarchic controller is applied to the beam design model: × open-loop poles; ○ closed-loop poles; + global design model poles.

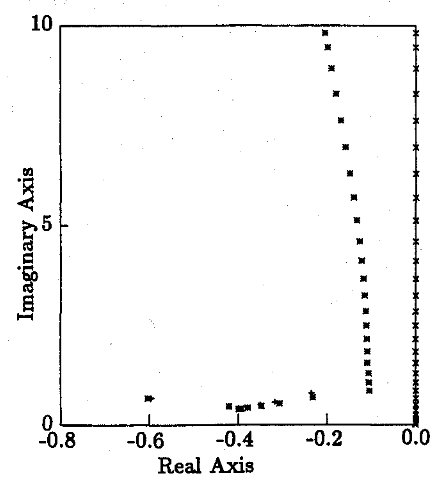

Fig. 6 Closed-loop poles when the hierarchic controller is applied to the beam design model using alternative 1: × open-loop; ∗ closed-loop poles; + global design model poles.

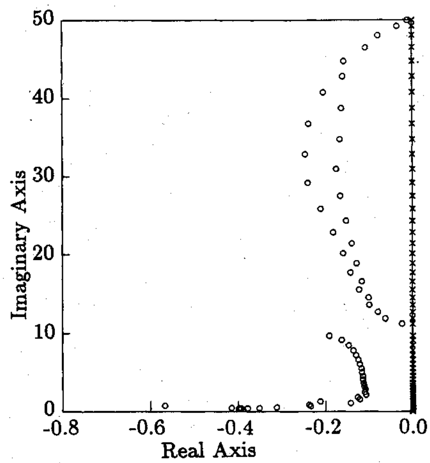

Fig. 7 Closed-loop poles when the hierarchic controller is applied to the beam evaluation model: × open-loop poles; ○ closed-loop poles.

Fig. 5. The open-loop poles refer to those of the condensed beam model with no control applied. The closed-loop poles are the poles of the condensed model with both of the global and local controllers applied. The global design model poles represent the closed-loop poles of the uncoupled global model of Eq. (43). For clarity, the closed-loop poles of the residual design model are not shown as they would overlay the higher frequency closed-loop poles.

The most important result to note in Fig. 5 is the relative closeness of the closed-loop and design model poles. Of course, the poles deviate somewhat from those predicted due to the stiffness coupling that was neglected in the design process. It is apparent that the deviation is worse for poles with frequencies that approach the boundary of the two design models. This is a result of the inaccuracies at higher frequencies in the reduced-order global model. Fortunately, the coupling is insufficient to cause instability in this case. The effects of the stiffness coupling may be reduced using the first alternative discussed in Sec. VI. We chose to retain $n_l = 8$ modes in the global model. The results are summarized in Fig. 6. Note the much improved agreement between the closed-loop poles and those predicted by the two design models. However, employing this alternative would result in a performance degradation since the higher frequency global design model poles are significantly removed from the optimal locations. This can be fixed by improving the local control design, the topic of current research.

The closed-loop poles obtained when the original hierarchic controller is applied to the evaluation model are shown in Fig. 7. They indicate that some spillover to the unmodeled higher frequency modes does occur, but the resulting closed-loop system is clearly stable.

The computational advantages of the hierarchic control architecture can be analyzed by performing an operations count for the control structure given in Fig. 3. One aspect of this architecture that is still under investigation is the best way to organize the communication between the local and global controllers. For this simple example, it will be assumed that perfect communication can be performed, but it is recognized that this is a strong idealization. In this analysis, the mass matrix is assumed to be a diagonal lumped approximation so that $T_g^T M$ is, in the same sense as T_g, block diagonal. The operations count can be obtained most easily by examining the steps in Fig. 3. To compute $T_g^T M q$, each local processor multiplies the nonzero block of $T_g^T M$ corresponding to that processor by the part of the vector q to which it has access. These partial results are passed up to the global processor, where they are summed to form the total result $T_g^T M q$. The global processor then multiplies this result by M_{gg}^{-1} to get q_g. The elements of q_g that correspond to each local processor are then passed back down by the global processor. The local processors then calculate that portion of $T_g q_g$ that they need to compute the residual e. This division of labor continues in a similar fashion through the remaining blocks of Fig. 3.

Table 1 Comparison of the equivalent multiplication requirements for hierarchic and full-state feedback controllers for a condensed model of a beam with $n = 90$ nodes

Controllers	Total number	Number at the global level	Number for a local region
Full-state feedback	21,570	21,570	0
Hierarchic			
$n_g = n_l = 32$	9853	6823	202
Hierarchic alternative 1			
$n_g = 32$, $n_l = 16$	7797	4767	202

To evaluate the computational burden of the hierarchic controllers, the number of operations required to implement the algorithm was calculated and compared to the results for full state feedback (Table 1). The results are given in terms of the number of equivalent multiplication operations since typical single-chip microprocessors require three times as long to perform a multiplication as an addition.[47] The results in the table are given for a beam with 90 nodes.

Looking at these results, it is clear that the hierarchic designs have an advantage in terms of the total number of operations required. However, what is more significant is that the workload for any individual processor is greatly reduced. In particular, each local processor is required to perform only about 2% of the total number of operations. This may allow the use of very simple, inexpensive local processing units. Alternatively, it may be possible to operate the residual control (which corresponds roughly to the higher frequency modes) at a much higher bandwidth. Note also the substantial savings at the global level. For comparison, implementing a decentralized controller designed as the local controller was in Eq. (48) requires only about the same number of operations as is necessary for each local region in the hierarchic design. However, the performance obtained when this type of feedback is implemented is quite poor since either the lower frequency poles are insufficiently damped to achieve this realistic objective, or the higher frequency poles are overdamped.

An analysis of smaller systems has indicated that there is a threshold for the system size above which the hierarchic control architecture offers similar performance but substantial computational savings over the full state feedback, and, for larger systems, the computational advantages become even more significant.

IX. Conclusion

This paper has presented the initial development of a hierarchic control technique for flexible structures. A two-level architecture was outlined that consists of many regional controllers and one central global controller. The hierarchic architecture was developed to reduce both the computational and input/output communication requirements when compared to using a single centralized processor for a system with a large number of sensors and actuators. Furthermore, the hierarchic control architecture is capable of yielding low interaction between the global and residual control functions. This interaction in the condensed model is due only to elastic coupling between the global and residual control design models and may be reduced or eliminated by an appropriate choice of interpolation functions. Of course, there will be other contributions to the spillover due to the errors in the evaluation model and the condensation procedure. The use of spatial filtering means that the interaction is small, so that the control laws for the global and residual models may be designed independently, which greatly simplifies the control synthesis process. The benefit of this hierarchic architecture is that it distributes the processing burden and reduces the computational requirements for controlling active structures while still allowing realistic control objectives to be achieved.

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